O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Spring 2008
Midterm Exam #2

Choose any four out of five problems.
Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____;

Name : ______________________________

E-Mail Address:_______________________________________

<u>Problem 1</u>:

Let

$$V^{\perp} = Span\left(\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -5 & 1 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}\right),$$

determine the original space, V. For $x = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$, find its direct sum representation of $x = x_1 \oplus x_2$, such that $x_1 \in V$, and $x_2 \in V^{\perp}$ (i.e., the direct sum of spaces V and $V^{\perp}$ is the set of all 2×2 matrics with real coefficients).

<u>Problem 2</u>:

Let $V = F^3$, and let F be the field of rational polynomials. Determine the representation of $v = \begin{bmatrix} s+2 & \frac{1}{s} & -2 \end{bmatrix}^T$ in (V, F) with respect to the basis $\{v^1, v^2, v^3\}$, where $v^1 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T$, $v^2 = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, $v^3 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T$.

Problem 3:

$$A = \begin{bmatrix} 3 & 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 & 0 \end{bmatrix}$$

What are the rank and nullity of the above linear operator, A ? And find the bases of the range spaces and the null spaces of the operator, A ?

Problem 4:

Consider the subspace of $\Re^4$ consisting of all 4×1 column vector $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ with constraints $x_1 + x_2 + x_3 = 0$ and $2x_1 + 2x_2 + 2x_3 = 0$. Extend the following set (with only one element) to form a basis for THE subspace:

$$\begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \end{bmatrix}.$$

<u>Problem 5</u>:

Show if the following sets

$$\begin{bmatrix} 3 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -2 \\ 2 \end{bmatrix}$$

span the same subspace V of $\left(\Re^4, \Re\right)$.